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Biomorphs via modified iterations

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Abstract

The aim of this paper is to present some modifications of the biomorphs generation algorithm introduced by Pickover in 1986. A biomorph stands for biological morphologies. It is obtained by a modified Julia set generation algorithm. The biomorph algorithm can be used in the creation of diverse and complicated forms resembling invertebrate organisms. In this paper the modifications of the biomorph algorithm in two directions are proposed. The first one uses different types of iterations (Picard, Mann, Ishikawa). The second one uses a sequence of parameters instead of one fixed parameter used in the original biomorph algorithm. Biomorphs generated by the modified algorithm are essentially different in comparison to those obtained by the standard biomorph algorithm, i.e., the algorithm with Picard iteration and one fixed constant.

Keywords: Biomorph, escape time algorithm, Mann iteration, Ishikawa iteration.
2010 MSC: 68T10, 68U05, 28A80.

1. Introduction

The fractals introduced by Mandelbrot [17] gained much attention in the computer graphics community because the obtained patterns were beautiful, very complex and were generated with the use of very simple formulas. Many different types of fractals were found: fractals generated through Iterated Function Systems [3], L-systems [25], strange attractors [31], complex fractals [3], etc.

In this paper we concentrate only on the complex fractals. Mandelbrot set, Julia sets, biomorphs, Newton fractals are typical complex fractals. All over the Internet one can find images of this type of fractals. Their images are source of inspiration for computer graphics artists and researchers. Many techniques and modifications were introduced to obtain these fractals. The techniques include: the inverse iteration method [7], the boundary scanning method [29], the level set method [22], the continuous potential method [6] and

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even the L-systems [21]. One of the widely used technique of obtaining the Julia sets is the escape time algorithm [3], and it is used in this paper. Rani et al.[27] introduced a modification of the Julia sets using the superior iterates. Sing et al.[30] by using also the superior iterates, introduced superior superfractals – a modification of Barnsley's superfractals [4]. In 2007 Ashlock et al.[2] presented generalized Julia sets obtained by modification of the constant in the Julia set generating algorithm.

In 1986 Pickover introduced biomorphs [23] – a modification of the Julia sets. Biomorphs were found accidentally as a programming bug [12]. Since their presentation in 1986 they were used by artists to produce aesthetic images [16]. Moreover, biomorphs found applications in biology. Mojica et al.[19] examined how unicellular organisms, as well as some specialized eukaryotic cells, evolve through a master plan of transformations under Darwinian natural selection from an initial population of cells simulated as Pickover biomorphs. And Levin [15] used biomorphs as an example of emergence of complex morphology from simple low-level rules. In the rest of the paper we present two modifications of the biomorph algorithm which will expand the class of biomorphs.

The paper is organized as follows. In Sec. 2 we introduce the notion of Julia set and the algorithm for generating its approximation (the escape time algorithm). Moreover, we present biomorphs and the generalized Julia sets. Next, in Sec. 3 we present different types of iterations, i.e., Picard, Mann, Ishikawa and two modifications of the biomorph algorithm. Then, in Sec. 4, we show some sample patterns obtained by using the proposed algorithm. Finally, in Sec. 5 we give some concluding remarks.

2. Julia Sets and Biomorphs

The name "Julia set" comes from the surname of a French mathematician Gaston Julia (1893-1978). His work about Julia sets appeared in 1918. The starting point of his work was a paper by a British mathematician Sir Arthur Cayley entitled "The Newton-Fourier Imaginary Problem" (1879).

Julia took a complex polynomial $f : \mathbb{C} \to \mathbb{C}$ of the form:

$$f(z) = z^2. \tag{2.1}$$

Then he iterated this function:

$$\begin{cases} f^0(z) = z, \\ f^n(z) = f(f^{n-1}(z)) + c, n \geq 1, \end{cases} \tag{2.2}$$

where $c \in \mathbb{C}$ is a given parameter. It turned out that for a given $z \in \mathbb{C}$ two possibilities can happen in the iteration process when n tends to infinity: iterations escape to infinity or they remain in a bounded area. The set of points for which the iteration process remains in a bounded area is called a filled Julia set and its boundary – a Julia set [3].

We can draw an approximation of Julia set using the escape time algorithm (see Algorithm 1). Some examples of Julia sets approximations are shown in Fig. 1 and the parameters used in the generation process of those sets are presented in Tab. 1.

In 1986 Pickover [23] introduced biomorphs (biological morphologies). He discovered them accidentally while writing a program for drawing approximation of the Julia set. In his program he made a mistake. Before drawing a point he added the following condition in Algorithm 1:

$$|\Re z| < R \vee |\Im z| < R, \tag{2.3}$$

where $\Re z$ and $\Im z$ are the real and imaginary parts of z, respectively. The images obtained with the help of his mistaken program were interesting and resembled single cellular organisms with internal structures – organelle. So, then, he also tried a few mappings other than the quadratic one [24], e.g.,

$$f(z) = z^z + z^6, \tag{2.4}$$

$$f(z) = \sin z + z^2, \tag{2.5}$$

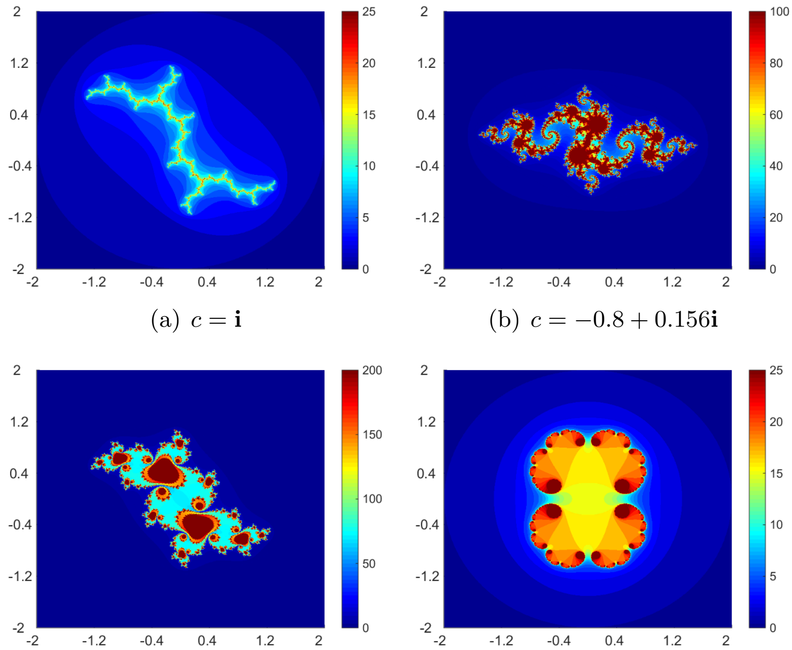
$$f(z) = \sin z + e^z. \tag{2.6}$$

Algorithm 1: Generation of Julia set approximation

Input: $f : \mathbb{C} \to \mathbb{C}$ – polynomial of degree greater than 1, $c \in \mathbb{C}$ – parameter, $K \in \mathbb{N}$ – maximum number of iterations, $R \in \mathbb{R}_+$ – threshold, $x_{min}, x_{max}, y_{min}, y_{max}$ – range of the area, $s \in \mathbb{R}_+$ – step.

Output: Approximation of a Julia set in a given area.

for $x = x_{min}$ **to** x_{max} *by* s **do**
    for $y = y_{min}$ **to** y_{max} *by* s **do**
        $z = x + y\mathbf{i}$
        $i_c = 0$
        for $i = 1$ **to** K **do**
            $z = f(z) + c$
            if $|z| > R$ **then**
                $i_c = i$
                break
        PrintDotAt(x, y) with color i_c



(a) $c = \mathbf{i}$ (b) $c = -0.8 + 0.156\mathbf{i}$

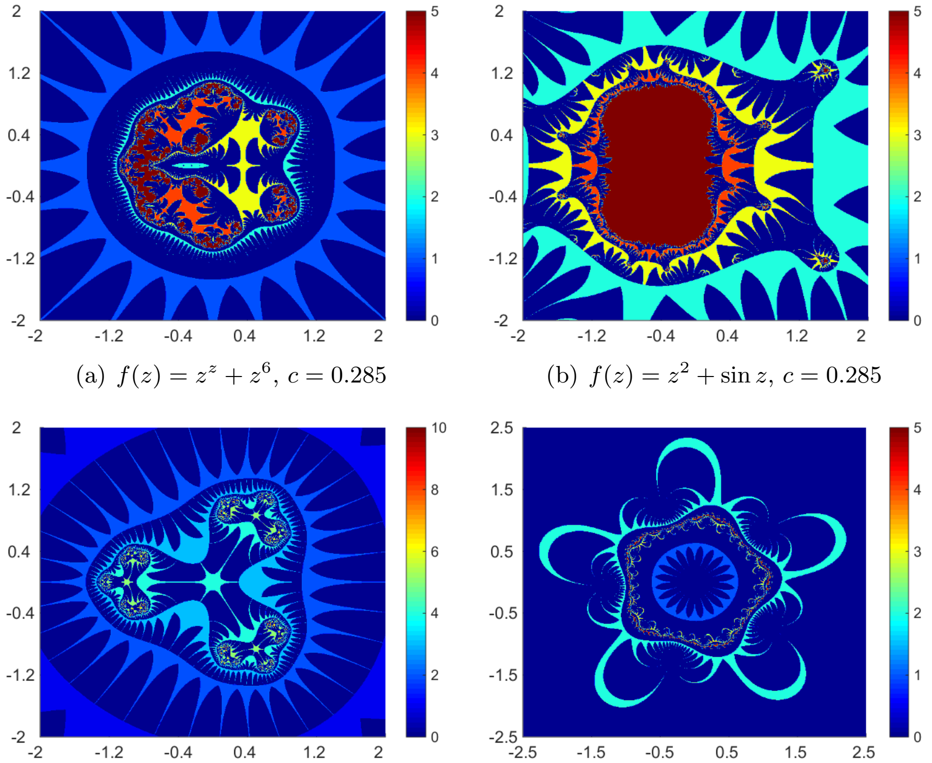
(c) $c = -0.1 + 0.651\mathbf{i}$ (d) $c = 0.285$

Figure 1: Examples of Julia sets

Table 1: Parameters of the Julia sets from Fig. 1

Julia set	c	K	R	Area	s
(a)	$\mathbf{i}$	25	2	$[-2, 2]^2$	$1/600$
(b)	$-0.8 + 0.156\mathbf{i}$	100	2	$[-2, 2]^2$	$1/600$
(c)	$-0.1 + 0.651\mathbf{i}$	200	2	$[-2, 2]^2$	$1/600$
(d)	0.285	25	2	$[-2, 2]^2$	$1/600$

Pickover was convinced that he discovered the Laws of Nature, which determine the appearance of living organisms. In this approach there was more mysticism than science. Despite of this, biomorphs certainly are graphic objects with very interesting shapes. By using different complex functions and parameters Pickover obtained a "zoo" of different forms of biomorphs that visually resembled primitive one-celled living forms. The creature's shapes obtained in this way resemble living organisms such as bacteria. It seems to be possible that proper manipulation of parameters provides opportunities to simulate details like cell wall or membrane, nucleus or other organelles in the generated image. Some examples of biomorphs are presented in Fig. 2 and the parameters used to generate those biomorphs are given in Tab. 2.



(a) $f(z) = z^z + z^6$, $c = 0.285$

(b) $f(z) = z^2 + \sin z$, $c = 0.285$

(c) $f(z) = z^3$, $c = 1$

(d) $f(z) = 1/z^5$, $c = 0.3241 - 0.08743\mathbf{i}$

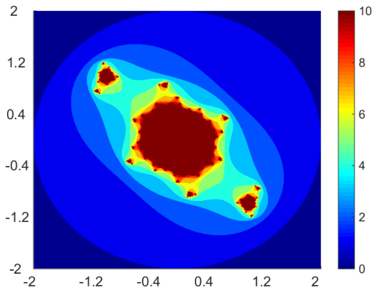
Figure 2: Examples of biomorphs

Table 2: Parameters of the biomorphs from Fig. 2

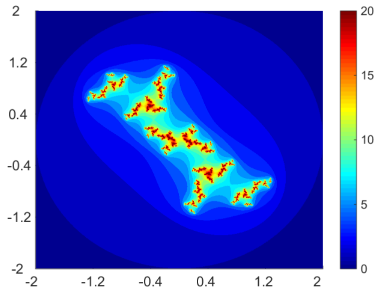
Biomorph	$f(z)$	c	K	R	Area	s
(a)	$z^z + z^6$	0.285	5	10	$[-2, 2]^2$	1/600
(b)	$z^2 + \sin z$	0.285	5	10	$[-2, 2]^2$	1/600
(c)	z^3	1	10	10	$[-2, 2]^2$	1/600
(d)	$1/z^5$	$0.3241 - 0.08743\mathbf{i}$	5	10	$[-2.5, 2.5]^2$	1/600

In 2007 Ashlock et al.[2] introduced another modification of the Julia set drawing algorithm. They used two parameters $c_1, c_2 \in \mathbb{C}$ instead of one c. In the odd steps of the iteration procedure they added c_1 and in the even steps c_2. By using this simple modification they obtained new shapes of Julia sets, and they called them generalized Julia sets.

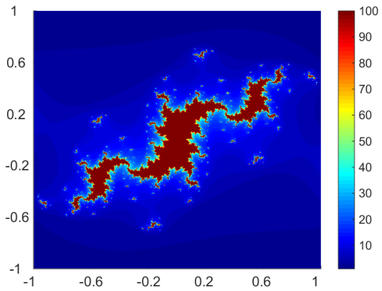
The examples of generalized Julia sets are presented in Fig. 3 and the parameters used to generate those sets are given in Tab. 3.



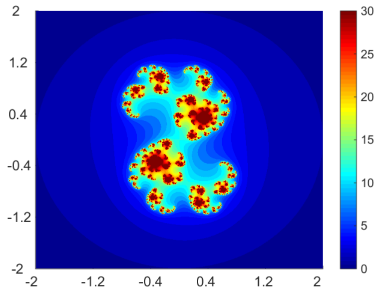
(a) $c_1 = \mathbf{i}$, $c_2 = 1$



(b) $c_1 = \mathbf{i}$, $c_2 = -0.1 + 0.651\mathbf{i}$



(c) $c_1 = -0.8 + 0.156\mathbf{i}$, $c_2 = -0.1 + 0.651\mathbf{i}$



(d) $c_1 = 0.45 + 0.1428\mathbf{i}$, $c_2 = 0.285$

Figure 3: Examples of generalized Julia sets

Table 3: Parameters of the generalized Julia sets from Fig. 3

Gen. Julia set	c_1	c_2	K	R	Area	s
(a)	$\mathbf{i}$	1	10	2	$[-2,2]^2$	$1/600$
(b)	$\mathbf{i}$	$-0.1+0.651\mathbf{i}$	20	2	$[-2,2]^2$	$1/600$
(c)	$-0.8+0.156\mathbf{i}$	$-0.1+0.651\mathbf{i}$	100	2	$[-1,1]^2$	$1/600$
(d)	$0.45+0.1428\mathbf{i}$	0.285	30	2	$[-2,2]^2$	$1/600$

3. Biomorphs via Different Types of Iterations

Let $w : X \to X$ be a mapping on a metric space (X, d), where d is a metric. Further, let $u_0 \in X$ be a starting point. Let us recall some popular iterative procedures [5]:

- Picard iteration:
$$u_{n+1} = w(u_n), \quad n = 0, 1, 2, \dots, \tag{3.1}$$
- Mann iteration:
$$u_{n+1} = \alpha_n w(u_n) + (1 - \alpha_n)u_n, \quad n = 0, 1, 2, \dots, \tag{3.2}$$
where $\alpha_n \in (0, 1]$,
- Ishikawa iteration:
$$\begin{aligned} u_{n+1} &= \alpha_n w(v_n) + (1 - \alpha_n)u_n, \\ v_n &= \beta_n w(u_n) + (1 - \beta_n)u_n, \quad n = 0, 1, 2, \dots, \end{aligned} \tag{3.3}$$
where $\alpha_n \in (0, 1]$ and $\beta_n \in [0, 1]$.

The standard Picard iteration is used in the Banach Fixed Point Theorem [5] to obtain the existence of the fixed point x^* such that $x^* = w(x^*)$. The approximation of this point is obtained under additional assumptions on the space X that it should be a Banach one and the mapping w should be contractive. The Mann [18] and Ishikawa [11] iterations allow to weak the assumptions on the mapping w. It is easily seen that the Ishikawa iteration with $\beta_n = 0$ is a Mann iteration, and with $\beta_n = 0$ and $\alpha_n = 1$ this is a Picard iteration. The Mann iteration with $\alpha_n = 1$ is a Picard iteration.

The Mann iteration with $\alpha_n = \alpha$, where $\alpha \in (0, 1]$, was successfully used in expanding the class of Julia sets [27], Mandelbrot set [28], superfractals [30] and in automatic generation of aesthetic patterns using dynamical systems [9]. Whereas the Ishikawa iteration with $\alpha_n = \alpha$, $\beta_n = \beta$, where $\alpha \in (0, 1]$, $\beta \in [0, 1]$, was used to obtain new patterns in polynomiography [14].

In Algorithm 1 the Picard iteration is used in the iteration process. Because the Ishikawa iteration is the most general one we use it in the iteration process for biomorphs as the first modification of the biomorph algorithm. In the Ishikawa iteration we can take arbitrary sequences α_n, β_n, but in our further considerations, for simplicity, we take $\alpha_n = \alpha$, $\beta_n = \beta$, such that $\alpha \in (0, 1]$ and $\beta \in [0, 1]$.

The second modification of the biomorph algorithm is based on Ashlocks' generalized Julia sets. We use a sequence of parameters d_n based on two parameters $c_1, c_2 \in \mathbb{C}\backslash\{0\}$ given by:

$$\begin{cases} d_0 = c_1, \\ d_{2n-1} = \frac{1}{c_1^{2n-1}} - d_{2n-2}, \quad n \geq 1, \\ d_{2n} = \frac{1}{c_2^{2n}} - d_{2n-1}, \quad n \geq 1. \end{cases} \tag{3.4}$$

The final algorithm of biomorph generation with our two modifications is presented in Algorithm 2. As we can see, with the help of the algorithm we are able to obtain biomorphs using:

- the standard Pickover's algorithm – $switch = false$, $\alpha = 1$, $\beta = 0$,
- the switching process – $switch = true$, $\alpha = 1$, $\beta = 0$,
- the Ishikawa iteration – $switch = false$, $\alpha \in (0, 1]$ and $\beta \in [0, 1]$,
- the switching process and Ishikawa iteration – $switch = true$, $\alpha \in (0, 1]$ and $\beta \in [0, 1]$.

4. Examples

In the first example we show the influence of changing parameters c_1, c_2 on biomorph's shape. In Fig. 4(a) the original biomorph is presented, obtained with the Pickover's algorithm using parameters: $f(z) = z^3$, $c = 1.5\mathbf{i}$, $K = 15$, $R = 10$, $s = 1/600$, the range of the area $[-2, 2]^2$. The other images in Fig. 4 were generated using Algorithm 2 with the same common parameters, and $\alpha = 1$, $\beta = 0$, $switch = true$ and:

(a) $c_1 = 1.5\mathbf{i}$, $c_2 = 1 + \mathbf{i}$,
(b) $c_1 = 1.5\mathbf{i}$, $c_2 = 0.285$,
(c) $c_1 = 0.7 - 0.35\mathbf{i}$, $c_2 = 1.5\mathbf{i}$,
(d) $c_1 = 0.285$, $c_2 = 1.5\mathbf{i}$.

From the obtained images we can see that the generated biomorphs differ in their shapes from the biomorph obtained with the Pickover's algorithm. Moreover, from (3.4) it is easy to see that when we take $-c_2$ instead of c_2 we obtain the same biomorph. From Fig. 4(c) and 4(e) we can see that if we exchange parameters c_1, c_2 with each other, then we obtain very different biomorphs.

In the next example we change parameter α in the iteration process. The original biomorph obtained with the help of Pickover's algorithm is presented in Fig. 5(a). The parameters used to generate this biomorph are: $f(z) = z^2 + \sin z$, $c = 1.234 + 0.032\mathbf{i}$, $K = 15$, $R = 10$, $s = 1/600$, the range of the area $[-3, 3]^2$. The other images in Fig. 5 were obtained with our algorithm using the same common parameters, and $\beta = 0$, $switch = false$ and:

(a) $\alpha = 0.8$,
(b) $\alpha = 0.6$,
(c) $\alpha = 0.4$,
(d) $\alpha = 0.2$.

Algorithm 2: Generation of Biomorph

Input: $f : \mathbb{C} \to \mathbb{C}$ – mapping, $c_1, c_2 \in \mathbb{C}\backslash\{0\}$ – parameters, $K \in \mathbb{N}$ – maximum number of iterations, $R \in \mathbb{R}_+$ – threshold, $x_{min}, x_{max}, y_{min}, y_{max}$ – range of the area, $\alpha \in (0, 1]$, $\beta \in [0, 1]$ – parameters for the Ishikawa iteration, $switch \in \{true, false\}$ – use the d_n sequence or not, $s \in \mathbb{R}_+$ – step.

Output: Biomorph.

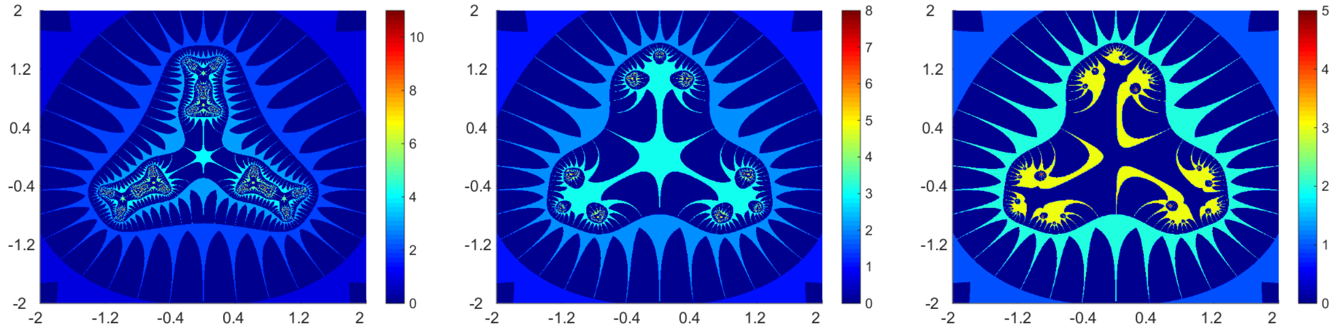
for $x = x_{min}$ **to** x_{max} $by\ s$ **do**
    for $y = y_{min}$ **to** y_{max} $by\ s$ **do**
        $z = x + y\mathbf{i}$
        $d = c_1$
        $i_c = 0$
        for $i = 1$ **to** K **do**
            $v = \beta(f(z) + d) + (1 - \beta)z$
            $z = \alpha(f(v) + d) + (1 - \alpha)z$
            if $|z| > R$ **then**
                $i_c = i$
                break
            if $switch = true$ **then**
                if $i \mod 2 = 0$ **then**
                    $d = \frac{1}{c_2^i} - d$
                else
                    $d = \frac{1}{c_1^i} - d$
        if $|\Re z| < R \vee |\Im z| < R$ **then**
            PrintDotAt(x, y) with color i_c
        else
            PrintDotAt(x, y) with color 0

From the images we can see that the shapes of the biomorphs differ from the original one. The change of parameter α causes that some parts of the biomorph separate and other parts join together forming a new biomorph. Because in Fig. 5 we can see only four images of new biomorphs we are not able to see that the shape of the biomorph changes in a continuous way with the continuous change of α. This change of the shape reminds an organism development process.

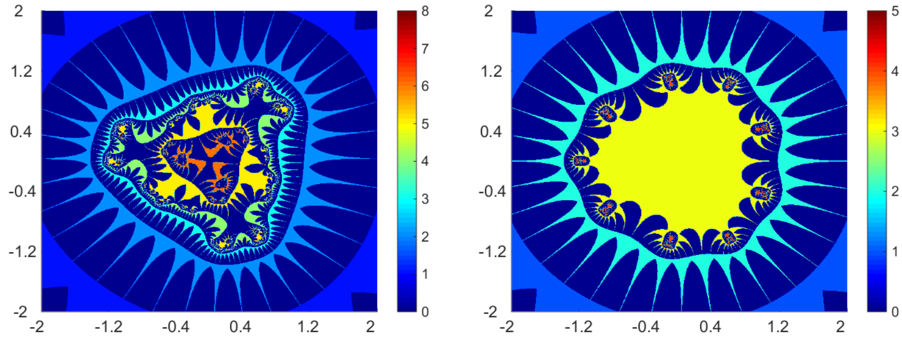
The third example is very similar to the previous one. This time we change parameter β with fixed parameter α in the iteration process. In Fig. 6(a) the original biomorph is presented, obtained with the following parameters: $f(z) = 2/3 \cdot (z^3 - 2)/z$, $c = 0.285$, $K = 15$, $R = 10$, $s = 1/600$, the range of the area $[-2, 2]^2$. The other images in Fig. 6 were obtained with the help of Algorithm 2 with the same common parameters, and $\alpha = 0.7$, $switch = false$ and:

(b) $\beta = 1.0$,
(c) $\beta = 0.8$,
(d) $\beta = 0.6$,
(e) $\beta = 0.4$.

Similarly to the previous example, the change of considered parameter (β in this case) results in change of biomorph's shape in comparison to the original one. Moreover, some parts of the biomorph separate and

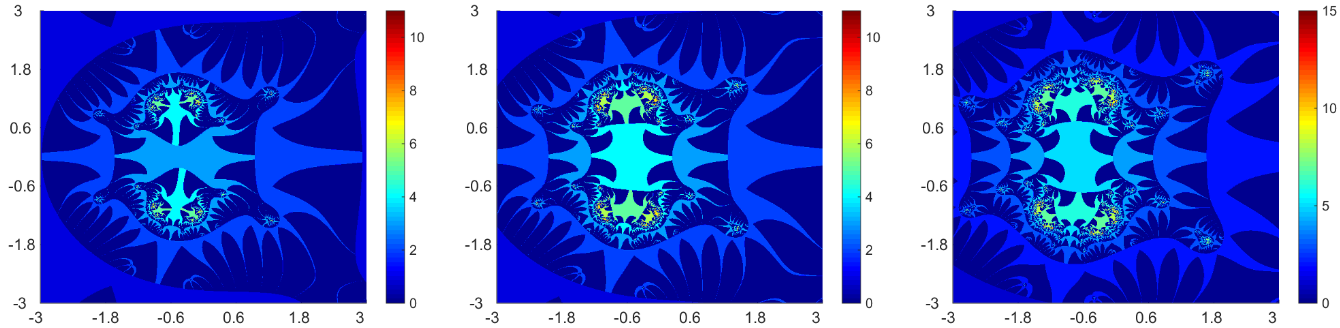


(a) original biomorph with $c = 1.5\mathbf{i}$ (b) $c_1 = 1.5\mathbf{i}$, $c_2 = 1 + \mathbf{i}$ (c) $c_1 = 1.5\mathbf{i}$, $c_2 = 0.285$

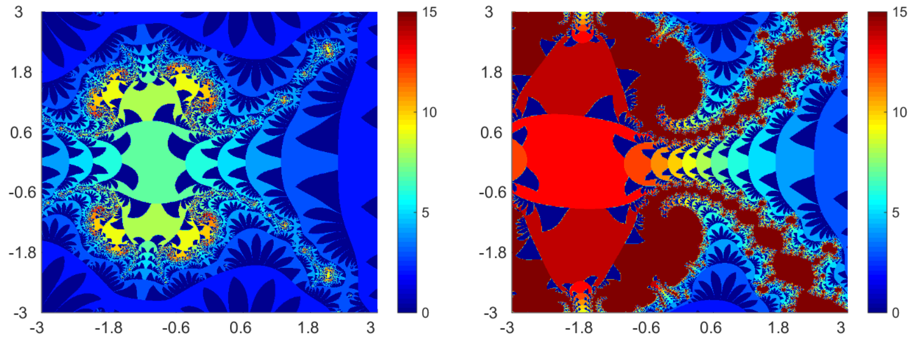


(d) $c_1 = 0.7 - 0.35\mathbf{i}$, $c_2 = 1.5\mathbf{i}$ (e) $c_1 = 0.285$, $c_2 = 1.5\mathbf{i}$

Figure 4: Examples of changing parameters c_1, c_2



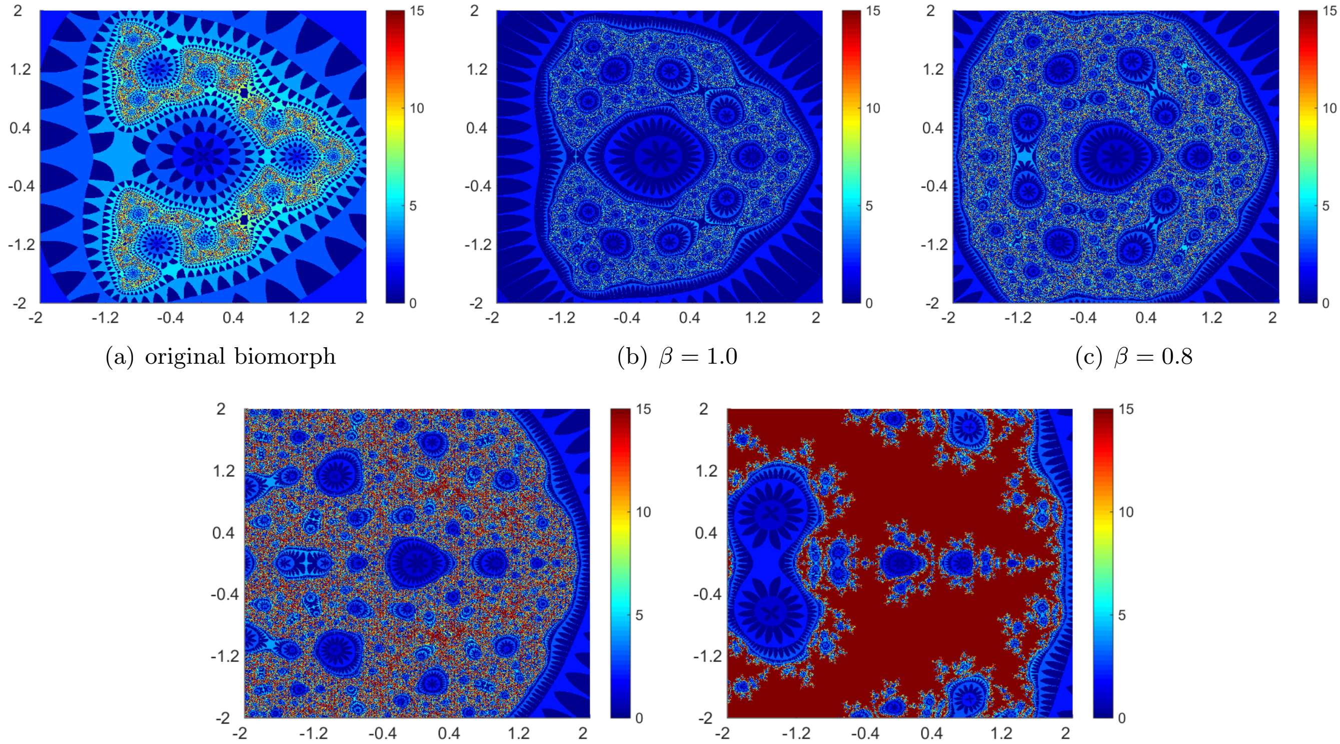
(a) original biomorph (b) $\alpha = 0.8$ (c) $\alpha = 0.6$



(d) $\alpha = 0.4$ (e) $\alpha = 0.2$

Figure 5: Examples of changing α parameter

other parts join together with the change of parameter β, forming a new biomorph. Like in the case of changing parameter α we do not see, from the images in Fig. 6, that for $\beta \in (0, 1]$ the biomorph's shape changes continuously with the continuous change of β.



(a) original biomorph (b) $\beta = 1.0$ (c) $\beta = 0.8$ (d) $\beta = 0.6$ (e) $\beta = 0.4$

Figure 6: Examples of changing β parameter

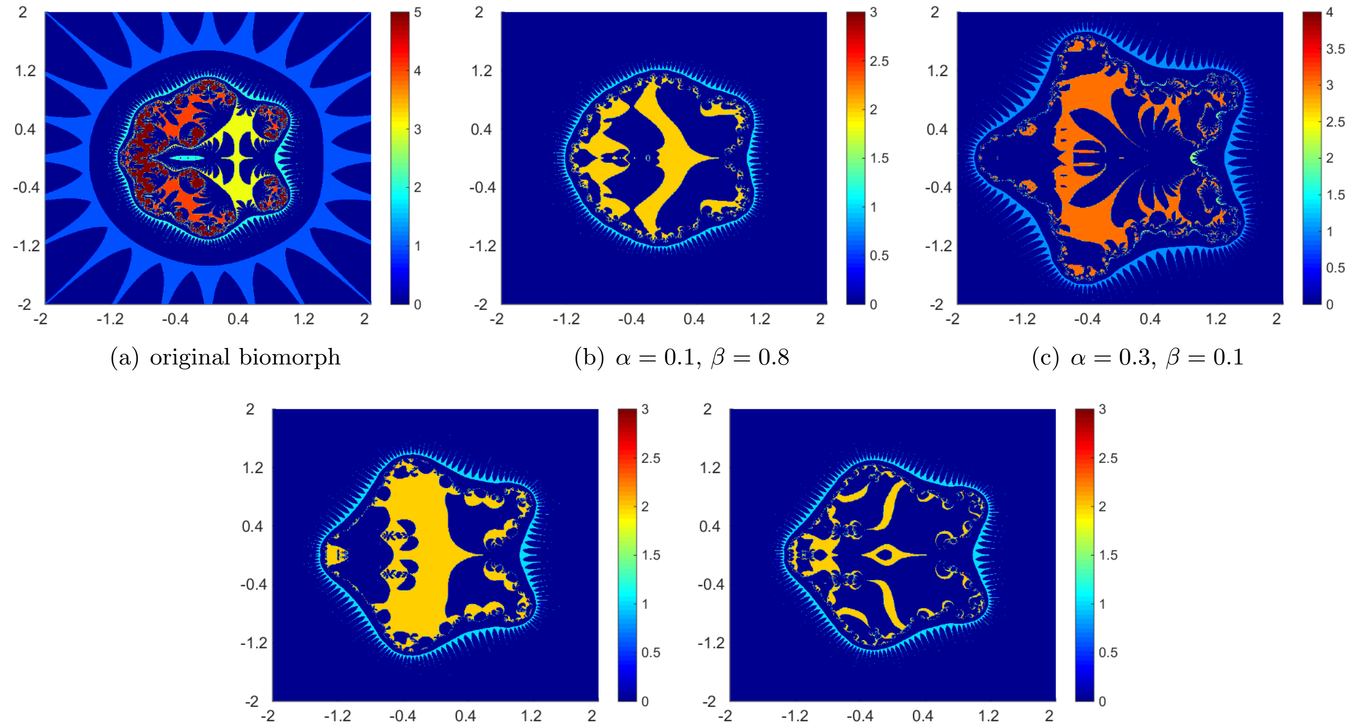
In the last example we show biomorphs obtained with our two modifications (the Ishikawa iteration and the sequence of constants) used together. In Fig. 7(a) there is presented the original Pickover's biomorph, obtained by using parameters: $f(z) = z^z + z^6$, $c = 0.285$, $K = 15$, $R = 10$, $s = 1/600$, the range of the area $[-2, 2]^2$. Figures 7(b)-(e) were obtained with our algorithm using the same common parameters, and $switch = true$, $c_1 = 0.285$, $c_2 = -0.285$ and:

(a) $\alpha = 0.1$, $\beta = 0.8$,
(b) $\alpha = 0.3$, $\beta = 0.1$,
(c) $\alpha = 0.8$, $\beta = 0.3$,
(d) $\alpha = 0.9$, $\beta = 0.4$.

Also in this case we can see very diverse biomorphs which differ from the original one.

5. Conclusions

In this paper we presented some modifications of the Pickover's biomorph generation algorithm. The modifications rely on the use of other types iterations, i.e., Mann or Ishikawa, instead of Picard iteration and on replacing one parameter by the sequence of parameters. The obtained biomorphs look quite different in comparison to those obtained with the help of the standard Pickover's algorithm. We believe that the results of the paper can be interesting for those whose works or hobbies are related to automatic creation of nicely looking graphics. Moreover, the modified biomorphs could also be used in extension of the research results conducted by Mojica et al.[19].



(a) original biomorph (b) $\alpha = 0.1$, $\beta = 0.8$ (c) $\alpha = 0.3$, $\beta = 0.1$

(d) $\alpha = 0.8$, $\beta = 0.3$ (e) $\alpha = 0.9$, $\beta = 0.4$

Figure 7: Examples of changing α, β parameters and switching the constants

In our further research we want to investigate how the change of the parameters influences the shapes of the biomorphs. This will give the ability of changing a biomorph's shape in a predictable way. Negi et al.[20] and Rani et al.[26] have made a research on the noise in superior Mandelbrot set and superior Julia sets, so we will try to extend their work on the modified biomorphs introduced in this paper.

When we search for an interesting biomorph we use the same technique like Pickover, i.e., we take very wide area, generate the biomorph and then narrow the area to the place where we found something interesting. So, we will try to develop an automatic biomorph searching method like those proposed by Ashlock et al.[1, 2]. We can also introduce the proposed modifications to other types of complex fractals and even to those based on quaternions.

The authors, basing on the papers on polynomiography [10, 13] and inversion fractals [8], also think that further progress in biomorphs is possible by using different non–standard types of iterations known in the fixed point theory. Changing the real and/or complex values of parameters in different types of iterations should essentially enlarge the set of biomorphs. Thanks to that it should be possible to find biomorphs' forms that really exist in the nature easier.

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